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Mathematical model of building envelope element insolation

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Abstract. In this paper, the problem of estimation of building envelope oriented element insolation by direct sunlight is solved using mathematical model of Earth's rotation. Quaternions are used as mathematical tool for description of rotation. The model allows to obtain automatically estimation of building element insolation by direct sunlight for given latitude and given time interval (month, week, day, etc.). The model takes into account schedule of change of day and night, change of direct sunlight angle on a given element of the surface caused by sun motion over horizon (changes of its height and azimuth during time of day). Distinctive feature of the model is simplicity of program-algorithmic implementation due to using description of rotation by means of quaternions.

1. Introduction

The level of solar insolation of building envelopes plays essential role in modern architectural design. The solar insolation factor is important for designing the forms of individual buildings, designing of sets and ensembles of buildings, as well as micro districts in cities as a whole. It requires the development of tools for estimating the level of solar insolation for building envelopes. Today, there are two main reasons for the necessity of such tools having in the design of building forms. On the one hand, it is necessary to provide sufficient inflow of sunlight, or vice versa, the level of shading for office, industrial and residential rooms of buildings in order to create comfortable conditions for their inhabitants [1]. On the other hand, it is necessary to take into account the development of renewable energy sources based on photovoltaic panels embedded in building envelopes (facades and roofs) – Building Integrated Photovoltaic (BIPV) [2-5]. For such technologies, the problem of optimal integration of PV elements into the building envelope, their orientation in accordance with insolation level are especially important, since their effectiveness directly depends on it. It is necessary to describe role and area of optimal application of BIPV technologies, which, according to the World Energy Council, is one of priority areas for the development of solar power in general [6]. The main reason for this is the intersection of trends in urbanization and trends in the development of electric power systems. As for urbanization, it is necessary to note the rapid development of cities, population growth and increasing of population density, it leads to construction of a large number of high buildings, increasing of buildings energy consumption. In some developed countries, the share of energy consumption by buildings is 35-40% of total energy consumption [6]. As for electrical power industry, one of the trends of development of power supply systems, in general, is the active usage of renewable energy sources, in particular, solar panels. The advantages of using solar generation to supplement existing power supply systems are: no need to build infrastructure for delivery of energy fuel (gas, diesel, coal, etc.); environmental friendliness, that is especially important in the conditions of modern city; constant reduction of cost of solar panels and components for systems with solar panels (controllers, inverters, energy storages); local generation, which increases the reliability of power supply for critical systems of buildings during power outages from external electrical grids, decreasing of load on the external power lines. The disadvantages of solar generation include instability (the level of solar irradiation on the surface of the Earth is not constant), large area that is needed to install panels. The first disadvantage can be partially overcome by the using of electricity storage devices; the second one is very difficult to be overcome in cities, because of absence and high



cost of land areas in cities for installation of arrays of solar panels. One of the approaches that can help to overcome these contradictions in urban conditions is integration of solar panels into the building envelopes - roof and facades - that is using of BIPV technologies. Aspects and information about the active development of BIPV technologies outlined above are supported by large number of various scientific works, performed using a variety of theoretical and experimental research methods, individual reviews [7], reports of international specialized agencies and committees [6, 8]. Separately, works can be noted that are dedicated to the development of specialized software for assessing insolation, not only in scope of individual separate buildings, but also in scope of entire ensembles and micro districts of cities [9, 10]. Additionally, it is necessary to note works about optimality of shapes of PV surfaces themselves [11]. Good illustrations and photos of buildings with BIPV are presented in [7, 11]. In addition, active work dedicated to regulation of BIPV technologies are described in [8]. All facts and number of papers speak about stability of this trend. As mentioned above, for BIPV technologies, the optimal orientation of solar panel arrays is particularly important from the point of view of direct sunlight falling on them, since the maximum generation of electricity, in the general case and for most solar panel types, is carried out when the sun's rays fall on the elements at 90° angle. In order to solve the problem of optimal integration of PV elements into the building envelope, mathematical models and algorithms are needed that give values of estimation of solar insolation. In this paper, an original method for estimating the insolation of the oriented element of the building envelope by direct sunlight is proposed. The core part of proposed method is quaternion model of Earth's rotation.

2. Quaternion model of Earth's rotation

In order solve problem of insolation we need to build model of Earth's rotation. Some simplifications can be taken due to problem has character of estimation. As base for model let us consider the following points:

1. Earth is considered as a ball;
2. Center of mass of Earth moves uniformly over circular orbit around the Sun, making full turn during 8760 hours;
3. Axis of Earth has a constant nutation angle 23.44 degrees;
4. Earth makes a full turn around its axis (self-rotation) during $24 \frac{365}{366} \approx 23.9344$ hours, which

corresponds to the complete revolution of the Earth around its axis relative to distant stars (It is «Sidereal day» in this model);

5. Earth period of rotation relative to Sun is 24 hours (period between two Sun culminations at one meridian);

6. Beginning of time in model is moment of the summer solstice for Greenwich (prime) meridian. Let us consider the orbital coordinate system $O\zeta\eta\xi$ with origins in Earth center of mass центр масс and following directions of axes: axis $O\zeta$ is directed opposite to Sun orbit movement, axis $O\eta$ is directed opposite to direction to Sun, axis $O\xi$ is perpendicular to ecliptic plane (figure 1). Introduced system $O\zeta\eta\xi$ is right-handed.

In this coordinate system Earth has regular precession with constant angular velocity ω_ψ , self-rotation with constant angular velocity ω_ϕ , constant angle of nutation:

$$\omega_\psi = -\frac{2\pi}{365 \times 24} \text{ rad/h}, \quad \omega_\phi = \frac{2\pi}{24 \times (365/366)} \text{ rad/h}, \quad \vartheta_0 = \frac{\pi}{180} \times 23.44 \text{ rad}.$$

Sign of minus in expression of ω_ψ is related to direction chosen axes of coordinate system.

For determination of orientation and position of objects on Earth surface we will use unit quaternion

$$\mathbf{q}(t) = \{q_0(t), q_1(t), q_2(t), q_3(t)\},$$

where $q_0^2(t) + q_1^2(t) + q_2^2(t) + q_3^2(t) = 1$.

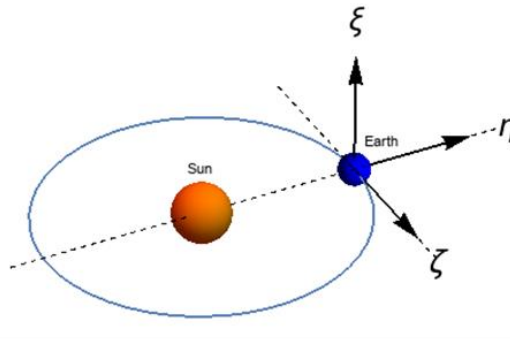


Figure 1. Illustration of orientation of introduced orbital coordinate system.

Taking into account precession movement of Earth in selected coordinate system $O\xi\eta\zeta$ components of quaternion of orientation are determined as [12]:

$$\begin{aligned} q_0(t) &= \cos \frac{\vartheta_0}{2} \cos \frac{(\omega_\psi + \omega_\varphi) t}{2}, & q_1(t) &= \sin \frac{\vartheta_0}{2} \cos \frac{(\omega_\psi - \omega_\varphi) t}{2}, \\ q_2(t) &= \sin \frac{\vartheta_0}{2} \sin \frac{(\omega_\psi - \omega_\varphi) t}{2}, & q_3(t) &= \cos \frac{\vartheta_0}{2} \sin \frac{(\omega_\psi + \omega_\varphi) t}{2}. \end{aligned}$$

Changing of position of any point related to Earth can be found using following rotation transformation:

$$\mathbf{r}(t) = \mathbf{R}(t) \mathbf{r}_0,$$

where \mathbf{r}_0 and $\mathbf{r}(t)$ - radius-vectors of initial and current positions of point in orbital coordinate system, and $\mathbf{R}(t)$ - matrix of rotation that is determined by components of quaternion:

$$\mathbf{R}(t) = \begin{pmatrix} 1 - 2q_2^2(t) - 2q_3^2(t) & -2q_0(t)q_3(t) + 2q_1(t)q_2(t) & 2q_0(t)q_2(t) + 2q_1(t)q_3(t) \\ 2q_0(t)q_3(t) + 2q_1(t)q_2(t) & 1 - 2q_1^2(t) - 2q_3^2(t) & -2q_0(t)q_1(t) + 2q_3(t)q_2(t) \\ -2q_0(t)q_2(t) + 2q_1(t)q_3(t) & 2q_0(t)q_1(t) + 2q_3(t)q_2(t) & 1 - 2q_2^2(t) - 2q_1^2(t) \end{pmatrix}.$$

3. Main orientations and schedule of changing of day and night

Let's take into consideration $\mathbf{a}(t)$ - vector of Earth axis (in direction from center of Earth to geographical North Pole), movement of this axis is determined by equation

$$\mathbf{a}(t) = \mathbf{R}(t) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Let us consider angle ϑ between normal vector to Earth's surface at given latitude λ of Greenwich (prime) meridian and Earth's axis

$$\vartheta = \frac{\pi}{2} - \frac{\pi}{180} \times \lambda,$$

here $\lambda > 0$ for Northern Hemisphere and $\lambda < 0$ for Southern Hemisphere.

Normal vector $n_\lambda(t)$ to Earth's surface at given latitude λ of Greenwich (prime) meridian is determined by equation

$$n_\lambda(t) = R(t)Q \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

where $Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \vartheta & -\sin \vartheta \\ 0 & \sin \vartheta & \cos \vartheta \end{pmatrix}$ - direction cosines matrix.

In this model we will consider direction to Sun sl to be constant for all points of Earth's surface (it is simplification but it is frequently used by reason of long distance between Earth and Sun and size of Sun). Vector sl according to selected coordinate system is determined as

$$sl = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}.$$

Visualization of positions and orientations of main vectors in selected orbital coordinate system at time zero point (summer solstice) is presented in figure 2.

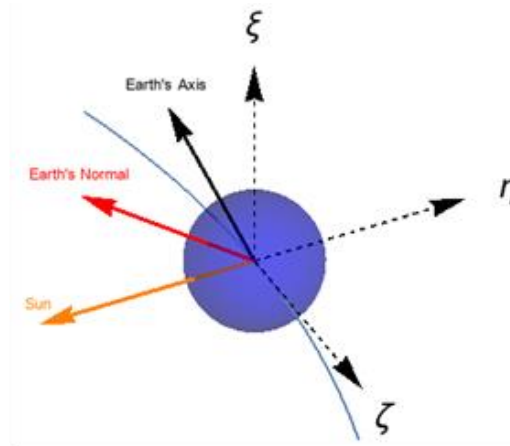


Figure 2. Visualization of main vectors (Earth's axis, normal vector at given latitude 60 degrees, vector directed to Sun) at summer solstice and orbital coordinate system.

The entered parameters and operations on them allow to obtain various information related to the movement of Earth. For example, the schedule of day and night, as well as the time of sunrise and sunset, can be determined by angle between the normal vector at a given point on surface of Earth $n_\lambda(t)$ and vector of direction to the Sun sl . In case if $c_1(t) = \langle n_\lambda(t), sl \rangle > 0$, there is day in this point on Earth surface, and if $c_1(t) = \langle n_\lambda(t), sl \rangle < 0$, then night. When $c_1(t) = \langle n_\lambda(t), sl \rangle = 0$, it corresponds to event of sunrise or sunset and depends on sign of expression of $\langle n_\lambda(t), sl(t) \rangle$ before.

4. Determination of insolation of building envelop element

Let us consider the local coordinate system $Oxyz$, that is constantly rigidly related to Earth surface in location of considered object (for example, considered building). Axis Ox is directed to East, axis Oy is directed to North, axis Oz is directed by normal vector to Earth surface (direction of axis Oz is the same as vector $n_\lambda(t)$). Orientations of local coordinate system $Oxyz$ and orbital coordinate system are demonstrated in figure 3.

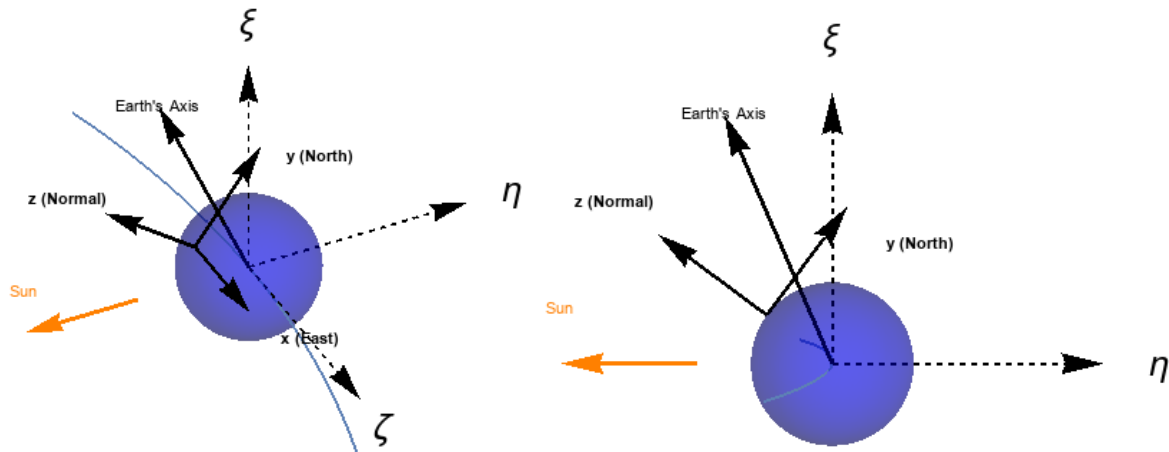


Figure 3. Local and orbital coordinate systems.

Let us consider normal vector to element of building envelope in local coordinate system:

$$n = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}.$$

Coordinates of this vector in orbital coordinate system are determined by transformation

$$\begin{pmatrix} n_\xi \\ n_\eta \\ n_\zeta \end{pmatrix} = Q \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix},$$

where Q is direction cosines matrix and it depends on given latitude (see above), in this case it is applied to go from local to orbital coordinate system.

The major part of insolation in clear sky day corresponds to direct sun irradiation [13]. Direct insolation of element of building envelope is determined by the variable geometric factor in form of cosine $c_2(t)$ of angle between the unit vector to Sun and unit vector normal of element of building envelope. This can be expressed in orbital coordinate system by expression

$$c_2(t) = \left\langle R(t) \begin{pmatrix} n_\xi \\ n_\eta \\ n_\zeta \end{pmatrix}, sl \right\rangle.$$

In order to solve comparative problems of estimation of insolation by direct sunlight, it is necessary to introduce a characteristic that takes into account changing of angle of incidence of direct sunlight on oriented element of building envelope over time. It should be taken into account that in some cases

when $c_2(t) > 0$, $c_1(t)$ may be even less than 0, i.e. Sun is lower than the horizon (for example, at winter solstice).

Taking into account discussion above we propose to use integral geometrical characteristic of insolation of oriented building envelop element in form of

$$I = \frac{1}{4} \int_{t_1}^{t_2} \left(1 + \frac{c_1(t)}{\sqrt{(c_1(t))^2}} \right) \left(1 + \frac{c_2(t)}{\sqrt{(c_2(t))^2}} \right) c_2(t) dt,$$

that describes insolation of selected unite element by direct sunlight on time interval (t_1, t_2) . Here $c_1(t)$ - cosine of angle between normal vector of Earth's surface and direction to Sun, it takes positive value when Sun is above the horizon; $c_2(t)$ - cosine of angle between normal vector of oriented element and direction to Sun, it takes positive value when angle between normal vector of oriented element and direction to the Sun is less than 90;

expression $\frac{1}{2} \left(1 + \frac{c_1(t)}{\sqrt{(c_1(t))^2}} \right)$ - gives 1 when angle between normal vector of Earth's surface and direction to Sun is less than 90 degrees, that is Sun is above the horizon, and gives 0 when it is night (it is a kind of «day indicator»);

expression $\frac{1}{2} \left(1 + \frac{c_2(t)}{\sqrt{(c_2(t))^2}} \right)$ - gives 1 when angle between normal vector of selected oriented element and direction to Sun is less than 90 degrees.

5. Results of numerical simulation and interpretation

In order to demonstrate work of proposed model and algorithm let us chose two conditional unit elements of building envelope – element of vertical facade oriented strictly in direction to geographical South Pole, and element of roof installed at angle of 45 degrees to surface of Earth and adjacent to facade under consideration (figure 4).

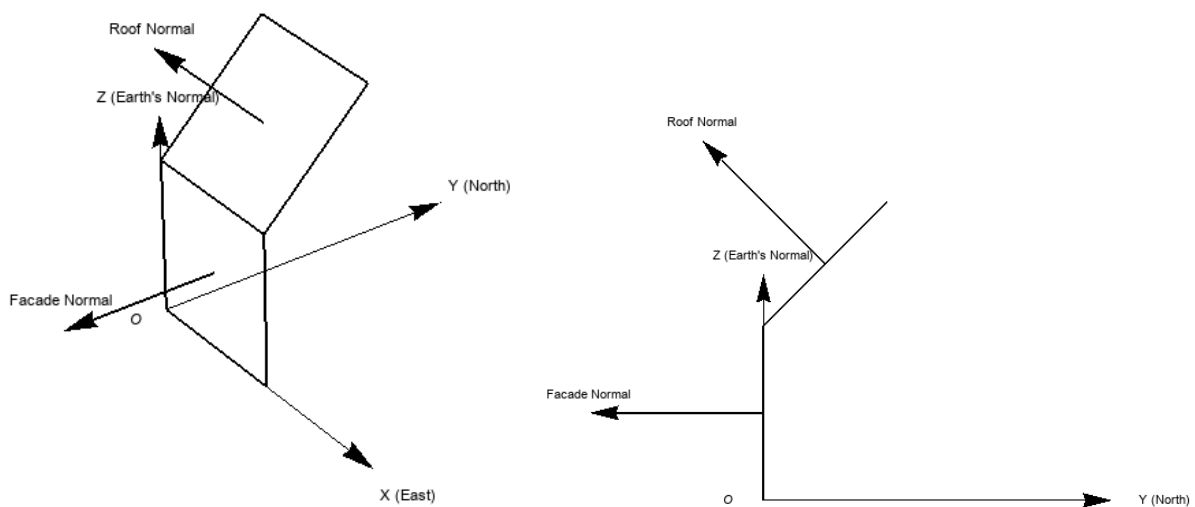


Figure 4. Illustration of orientations of vertical element of facade element of element of roof and their normal vectors in local coordinate system.

In local coordinate system $Oxyz$ normal vectors of selected elements are

$$n_{Facade}^{Oxyz} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, n_{Roof}^{Oxyz} = \begin{pmatrix} 0 \\ -\sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix}.$$

In orbital coordinate system they are

$$n_{Facade}^{O\zeta\eta\xi} = Qn_{Facade}^{Oxyz}, n_{Roof}^{O\zeta\eta\xi} = Qn_{Roof}^{Oxyz}.$$

Illustration of orientations of vertical element of facade element of element of roof and their normal vectors in local coordinate system $Oxyz$ is presented in figure 4. Visualization of the same vectors in orbital coordinate system $O\zeta\eta\xi$ is presented in figure 5.

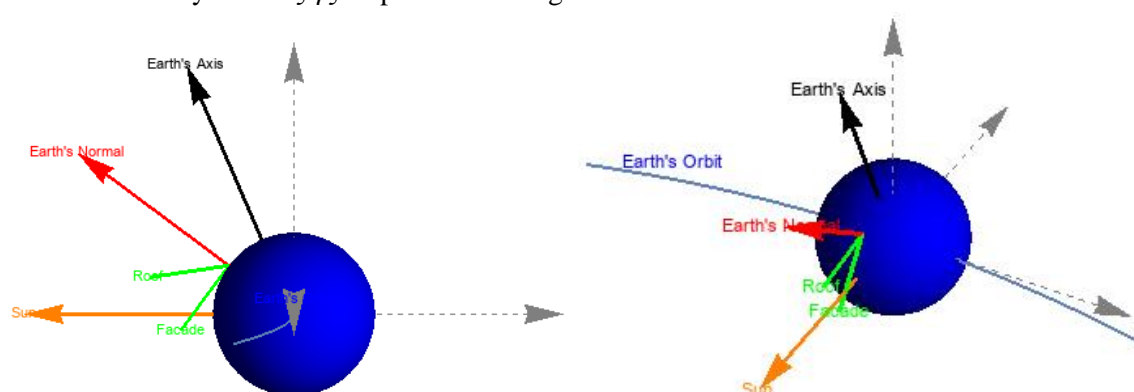


Figure 5. Illustration of orientations of normal vectors of vertical facade element and roof element in orbital coordinate system.

Now it is possible to calculate values of insolation integral I for facade and roof elements during intervals of four days - day of summer solstice (0 day), 91 days later (close to the autumn equinox), 182 days later (close to winter solstice), 274 days later (close to the spring equinox). Location is set at latitude of 60 degrees in Northern Hemisphere. The interval of integration is 24 hours. The results of numerical simulation are shown in table 1.

Table 1. Results of numerical simulation.

	0 day	+ 91 days	+ 182 days	+ 274 days
Facade	3.8756	6.6042	5.1038	6.6042
Roof	8.0534	7.3828	3.9032	7.3828

The insolation values for +91 days point and for +274 days point are equal, which is explained by the fact that planet occupies diametrically opposite positions in orbit according to the model ($365-91 = 274$). Insolation of facade in winter (+182 days point) is higher than insolation of roof, it is explained by lower height of Sun above the horizon in winter.

6. Conclusions

Proposed kinematic model of Earth rotation based on quaternions and proposed dimensionless characteristic I , that takes into account geometrical factor of direct sunlight inclination within selected time interval, allow to solve wide range of problems of comparative estimations. The proposed model is quite simple for implementation in mathematical simulation packages, such as MathCAD or Wolfram Mathematica, for example. The vector determination of positions of points on Earth's surface, orientations of main axes and elements of building envelope open wide opportunities for various visualization and solving of other related tasks.

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